Intro to Nuclear and Particle Physics (5110)

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Beta Decays
Beta Decays

• Beta decay is the process by which complex nuclei return towards the line of stability by emitting electrons or positrons, OR by electron capture:

\[
(Z, A) \rightarrow (Z + 1, A) + e^- + \bar{\nu}_e
\]

\[
(Z, A) \rightarrow (Z - 1, A) + e^+ + \nu_e
\]

\[
(Z, A) + e^- \rightarrow (Z - 1, A) + \nu_e
\]  
(electron capture)
Wek Interactions

But beta decay is just one example of “weak (nuclear)” force at work:

Some weak interaction decays involve only hadrons (things made of quarks)

\[ K^+ \rightarrow \pi^+ + \pi^0 \]

Some Decays or interactions involving leptons (things not made of quarks) alone.

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]

Beta decay is a “semi-leptonic decay” since in this type of decay both leptons and hadrons are all involved.

\[ n \rightarrow p + e^- + \bar{\nu}_e \]
A quick Preview of Fundamental Particles

- Notice the **two columns** of fundamental particles (though usually you see this diagram rotated 90 degrees).
- Notice the two columns differ in charge by 1 (e).
- The proton can be thought of as a “composite” particle on the right side, the neutron on the left.
- The **math** of the right/left is identical to that of spin-up, spin-down for a spin ½ particle: Called “Isospin”.

Form hadrons
Beta Decay (Continued)

The apparent process in \( \beta^- \)-decay is the conversion of nucleus \((Z,A)\) into nucleus \((Z+1,A)\) and an electron \((e^-)\):

\[
(Z, A) \rightarrow (Z + 1, A) + e^-
\]

It can be thought of as the conversion of a bound neutron \((n)\) into a bound proton \((p)\).

\[
\text{n} \rightarrow p + e^-
\]

\(\beta^-\) - decay

For some proton-rich nuclei they can frequently undergo \(\beta^-\)-decay in which a positron \((e^+)\) is emitted.

\[
p \rightarrow n + e^+
\]

\(\beta^+\) - decay

A proton-rich nucleus can capture an atomic electron and thereby change a proton into a neutron.

\[
e^- + (Z, A) \rightarrow (Z - 1, A) \quad \text{or} \quad e^- + p \rightarrow n
\]

\(\text{electron capture}\)

The electron is captured usually from the K***-shell but can be from the L, M, N, or even higher shell. In this process the electron in annihilated.

*** Recall K,L,M,N…etc \(\leftrightarrow n = 1,2, 3,4 \ldots\)
Beta Decay “Kinematics”


\[ {}^{64}\text{Cu}_{35} \rightarrow {}^{64}\text{Zn}_{34} \quad (38\%) \]

\[ {}^{64}\text{Cu}_{35} \rightarrow {}^{64}\text{Ni}_{36} \quad (19\%) \]

\[ {}^{64}\text{Cu}_{35} + e^- \rightarrow {}^{64}\text{Ni}_{36} \quad (43\%) \]
Beta Decay Mystery!

• The e- or e+ should be mono-energetic in a 2-body decay
• But the kinetic energy spectrum of the emitted electrons is continuous.
• Even worse: Consider the following decay. It seems to violate the law of angular momentum conservation.

\[ ^{14}_6\text{C} \rightarrow ^{14}_7\text{N} + \text{e}^- \]

This is the (in)famous carbon-14 isotope that is used for “dating”

\[ J = 0 \quad 1 \quad 1/2 \quad ? \]

• There is no way that angular momentum can be conserved in the decay:
  – Combination of \( J_1=1 \) and \( J_2=1/2 \) gives total \( J=1/2 \) or \( 3/2 \), but never 0
• Some, like Neils Bohr, declared that Classical Physics was Dead: that Energy and Angular Momentum are not conserved in the Quantum World!
In 1930 Wolfgang Pauli made a hypothesis that provided a solution to these difficulties and that has satisfied all experimental tests.

He proposed that an electrically neutral particle of spin 1/2 is created and emitted at the same time as the electron (or positron) in $\beta$-decay.

The particle is now called a neutrino (symbol, $\nu$) and it can take a share of the energy because in $\beta$-decay there is now a three-body configuration of the final state.

It turns out that nature has three kinds of neutrinos: one for each “lepton”

$$(\nu_e, \bar{\nu}_e) \quad (\nu_\mu, \bar{\nu}_\mu) \quad (\nu_\tau, \bar{\nu}_\tau)$$

1. The $\nu_\tau$ was only directly observed in 2000 by the DONUT Experiment.
2. Neutrinos have mass and oscillate (SuperK ~1998 + SNO ~2002)
We need to label the neutrinos in nuclear \( \beta \)-decay in the following fashion:

\[
\begin{align*}
\beta^- - \text{decay} & \quad (Z, A) \rightarrow (Z + 1, A) + e^- + \bar{\nu}_e \\
\beta^+ - \text{decay} & \quad (Z, A) \rightarrow (Z - 1, A) + e^+ + \nu_e \\
\text{Electron capture} & \quad e^- + (Z, A) \rightarrow (Z - 1, A) + \nu_e
\end{align*}
\]

In order to consider the energetics of \( \beta \)-decay, we need to know the mass of the neutrino emitted in \( \beta \)-decay. It is known to be less than 18 eV and since this is small compared to the total energy released in most \( \beta \)-decays we shall assume that the mass is zero. In fact the neutrino mass is of considerable significance in cosmology and in theories of elementary particles.
Free nucleon decays:

(1). A free neutron undergoes $\beta$ -decay with a mean life time $\tau = 898 \text{ s.}$

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

\[
Q_\beta = [M_n - (M_p + m_e)]c^2 \\
= [939.573 - (938.791 + 0.511)] \text{ MeV} \\
= 0.782 \text{ MeV} > 0
\]

This process is certainly **energetically possible** since $Q_\beta = 0.782 \text{ MeV}$

which is larger than zero.

(2). Consider the case for a free proton:

\[ p \rightarrow n + e^+ + \nu_e \]

\[
Q_\beta = [M_p - (M_n + m_e)]c^2 \\
= [938.791 - (939.573 + 0.511)] \text{ MeV} \\
= -1.293 \text{ MeV} < 0
\]

This process is energetically impossible since $Q_\beta = -1.293 \text{ MeV}$

which is smaller than zero.

This is a fortunate situation as the stability of protons (on the time scale of $>> 10^{14}$ years) is essential to the existence of the universe and of ourselves.
(3). Electron capture in a hydrogen atom

\[ e^- + p \rightarrow n + \nu_e \]

\[ Q_\beta = [(M_p+m_e) - M_n]c^2 \]
\[ = [(938.791+0.511) - 939.573] \text{ MeV} \]
\[ = -0.271 \text{ MeV} < 0 \]

This process is unlikely to happen as its \( Q_\beta \) value is seen smaller than zero. Furthermore the safety of the proton against electron capture follows immediately from the existence of free neutron decay.
Energy conditions in $\beta$-decay and electron capture in terms of nuclear masses, $M(Z,A)$:

\[
\begin{align*}
\beta^- : \quad (Z, A) &\Rightarrow (Z + 1, A) + e^- + \bar{\nu}_e \\
\beta^+ : \quad (Z, A) &\Rightarrow (Z - 1, A) + e^+ + \nu_e \\
\text{EC} : \quad (Z, A) + e^- &\Rightarrow (Z - 1, A) + \nu_e
\end{align*}
\]

\[
Q_\beta = [M(Z,A) - M(Z + 1, A) - m_e]c^2 \quad > 0
\]

\[
Q_\beta = [M(Z,A) - M(Z - 1, A) - m_e]c^2 \quad > 0
\]

\[
Q_{EC} = [M(Z,A) - M(Z - 1, A) + m_e]c^2 \quad > 0
\]

If a condition is satisfied, then the appropriate decay is possible and the excess energy available is shared as kinetic energy among the products in a manner which conserves linear momentum.

Fig. 5.6 The energy-level diagrams for $\beta^-$ - $\beta^+$-decays and for electron capture (E.C.). Here it is most convenient to use a vertical scale which gives the atomic masses of the levels involved. In (a) the parent level has to be above the level of the daughter for $\beta$-decay to be possible, the level difference, $Q_\beta$, being the energy available to share among the products as kinetic energy, which, neglecting nuclear recoil, will be the maximum kinetic energy the electron can have. In (b) electron capture can occur and the mass difference ($Q_{EC}$) goes into total energy of the neutrino and recoil of the daughter atom (branch labelled E.C.). For $\beta^+$-decay to occur the mass difference must be greater than $2m_e$; what is left is available for kinetic energy ($Q_\beta$). This situation is represented by the right-hand branch of (b), labelled $\beta^+$. For an atomic mass difference less than $2m_e$, $\beta^+$-decay is impossible and only electron capture can occur, as shown in (c).

Note that electron capture can sometimes occur when $\beta^+$-decay is impossible.
Important Differences between alpha and beta decays

• In $\beta^-$ decay there is no potential barrier to penetrate (potential energy is negative)
• For $\beta^+$ decays the exponential Gamov factor $e^{-G} \sim 1$
• The electron and the anti-neutrino do not exist before the decay occurred → must account for the creation of 3 new particles
• The electron and neutrino mass are SO SMALL that even in nuclear beta decays we must treat them relativistically
• We must account for the continuous kinetic energy distribution of the decay electron in any model that seeks to explain beta decays:
  Fermi’s Theory of Beta Decay
Some more Quantum Mechanics

In order to describe the Fermi theory which successfully explains continuous β-decay spectra (both for β⁻ and β⁺ decays) we need to know the Fermi’s Golden rule.

A true stationary state (i.e. a solution of the time-independent Schrodinger equation) lives forever. The expectation values of physical observables, computed from the wave function of a stationary state, do not change with time.

In particular, the expectation value of the energy is constant in time. The energy of the state is precisely determined, and the uncertainty in the energy, ΔE vanishes.

\[ \Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2} \]

The Heisenberg relation, \( \Delta E \Delta t \geq \hbar/4\pi \), then implies that \( \Delta t = \infty \).

Thus a state with an exact energy lives forever; its lifetime against decay is infinite.
Suppose that a quantum mechanical system is subject to a small *perturbing* potential $V'$, in addition to the original potential $V$ ($V' \ll V$). In the absence of $V'$, we can solve the Schrödinger equation for the potential $V$ and find a set of eigenstates $\Psi_n$ and corresponding eigenvalues $E_n$.

If the small additional potential $V$ is included then the states are approximately, but not exactly, the previous eigenstates $\Psi_n$ of $V$.

This small additional potential permits the system to make transitions between the “approximate” eigenstates $\Psi_n$.

\[
\Psi_i \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \qua
A nonstationary state has a nonzero energy uncertainty $\Delta E$. This quantity is often called the “width” of the state and is usually represented by $\Gamma$.

The lifetime $\tau$ of this state can be estimated from the uncertainty principle by associating $\tau$ with the time $\Delta t$ during which we are permitted to carry out a measurement of the energy of the state.

Thus $\tau \approx \hbar / \Gamma$

The decay probability or transition probability $\lambda$ is inversely related to the mean lifetime $\tau$.

$$\lambda = \frac{1}{\tau}$$
If we have the following knowledge then it would be possible to calculate $\lambda$ or $\tau$ directly from nuclear wave functions.

1. The initial and final wave functions $\Psi_i$ and $\Psi_f$, which we regard as approximate stationary states of the potential $V$.

2. The interaction $V'$ that causes the transition between states.

The calculation of $\lambda$ is too advanced for even most graduate courses, but most undergraduate nuclear physics courses use the result, which is known as the **Fermi’s Golden Rule**:

$$\lambda = \frac{2\pi}{\hbar} |V_{fi}'|^2 \rho(E_f)$$

- The quantity $V_{fi}'$ has the form: $V_{fi}' = \int \psi_i^* V' \psi_i \, dv$ (overlap integral)
- The quantity $\rho(E_f)$ is known as the density of final states. If there is a larger density of states near $E_f$, there is a larger transition probability.
Density of States???

• OK…let’s use a simple example: imagine an excited hydrogen atom in a 7p state
  – By conservation of angular momentum, you can emit a photon (which has spin 1) only if \( \Delta L = 1 \)
  – So your choices are to decay to s or d states
  – But for a given \( n \), there is only one s state, but 5 d states…
  – So then the transition probability (per unit time) is roughly five times for the transition to d state than to s state.

• But the n=1,2 energy levels do not have d (L=2) states
  – So \( \frac{dn}{dE} \sim \frac{1}{(E_2-E_1)} \) at \( E \sim E_1 \) to \( E_2 \)

• But the n=3 state has 1s and 5 d states = 6 states
  – \( \frac{dn}{dE} \sim \frac{6}{(E_4-E_3)} \) at \( E \sim E_3 \) to \( E_4 \)
  – We are making the approximation that there are so many states we can treat them as being continuously distributed