Homework 3

(I) Trigonometry:

(a) *[7 pts.]* A vertical pole is 15 m high. At a given time, it's shadow on the horizontal field is measured to be 26 m long. How high is the Sun above the horizon at that time?

(b) *[7 pts.]* The same pole as in the previous question is now standing vertical in a street that runs north-south. The Sun culminates due south 60° above the horizon. Knowing that the street goes up southward with a 10% slope, what is the length of the pole's shadow at the time of Sun culmination?

(c) A little bit of geometry

i. *[2 pts.]* A parallelogram has its sides of length a and b forming and acute angle θ. Express the area of this parallelogram.

ii. *[2 pts.]* What is the area of a unit inner-radius hexagone ?

iii. *[2 pts.]* What is the area of a unit outer-radius hexagone ?

iv. *[2 pts.]* What is the circumference of a unit inner-radius octogone? *(Hint: Use the fact that* $\sin(\frac{\theta}{2}) = \sqrt{\frac{1-\cos(\theta)}{2}}$*)*

v. *[2 pts.]* What is the circumference of a unit outer-radius octogone? *(Hint: the same hint applies)*

vi. *[2 pts.]* Using the previous two questions to provide a bracketing of the value of π.

vii. *[2 pts.]* While we are at it, you know that the sum of the inner angles of a triangle amounts to π radians. What is the sum of the inner angles of a hexagone? What about an octogone?

(II) Calculus:

Find the expression of the first and second derivatives with respect to x of the following functions.

(a) *[5 pts.]* $f(x) = 3x^4 - 2x^3 + 3x^2 - 6x + 12$

(b) *[5 pts.]* $g(x) = \frac{1}{2-x^2}$

(c) *[5 pts.]* $h(x) = \frac{1}{(2-3x)^2}$

(d) *[5 pts.]* $f(x) = \frac{2-x^2}{(2-3x)^2}$

(e) *[5 pts.]* $g(x) = \cos(3x)$

(f) *[5 pts.]* $h(x) = \cos(x)\cos(2x) - \sin(x)\sin(2x)$

(g) *[5 pts.]* $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

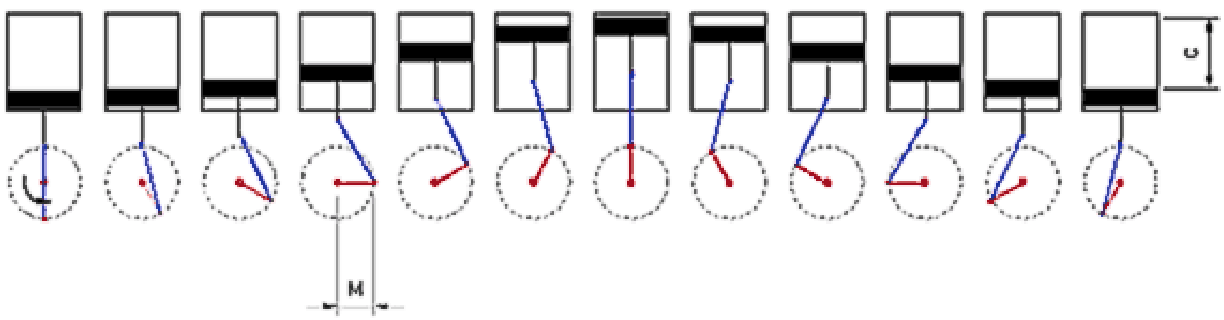
(h) *[5 pts.]* $g(x) = \sin(x)\cos(x)$

(i) *[5 pts.]* $h(x) = \sin(3x - 5x^2)$

(j) *[5 pts.]* $f(x) = \frac{2\sin(x)\cos(2x)}{\sin(3x^2)}$ (Only the first derivative for this one, the second derivative is too long and boring to obtain while not showing anything new.)

(III) Don't you feel cranky yet?

The following figure represents a piston connected to a crank. The radius of the crank (the length of the red line) is be R and the length of the connecting rods (the length of the blue line) is L. Suppose the wheel is turning at a constant angular velocity ω. The position of the wheel at a given time can be indicated by an angle $\theta(t)$ where t represents time. We can use the leftmost picture as a starting point with $t = 0$ and $\theta(0) = 0$ in such a way that $\theta(t) = \omega t$. The position of the piston as a function of time is denoted $x(t)$. As a reference we choose $x(0) = 0$ and $x(t)$ increases when the piston moves up on the figure.



The full cycle of the operation of a crank and piston assembly (Taken from the wikipedia page on crankshafts)

(a) *[8 pts.]* Find the expression of $x(t)$.

(b) *[5 pts.]* Find the expression of $v(t)$ the velocity of the piston.

(c) *[5 pts.]* Find the expression of $a(t)$ the acceleration of the piston.

(d) *[4 pts.]* Under what limit is the motion of the piston described by a simple harmonic function of time (sine or cosine).