Homework 5

(I) *[10 pts.]* A train goes at a speed of 108 km/h. The wheels are turning at an angular velocity of 40 rad/s. What is the diameter of the wheels?

(II) *[10 pts.]* You are standing right next to rectilinear train tracks. A train goes by at a speed of 108 km/h. As the engine passes you, the conductor blows the whistle. Exactly five minutes later, the engineer blows the whistle again. The speed of sound is 300 m/s. What is the time elapsed between your first hearing of the whistle and your second hearing of the whistle?

(III) *[10 pts.]* Let M be the mass of the Earth, R the radius of the Earth, g the gravitational acceleration at the surface of the Earth, and G the universal constant of gravitation. With M, L, and T representing physical dimensions of mass, length and time respectively, we have $[M] = M$, $[R] = L$, $[g] = LT^{-2}$, and $[G] = L^3M^{-1}T^{-2}$. Using dimensional analysis, give your best guess as to the what the expression of g in terms of M, R and G might be.

(IV) The gravitational acceleration on the Earth is $10\,\mathrm{m/s^2}$ while on the Moon it is only $2\,\mathrm{m/s^2}$. Some experiment involves launching an object vertically with some initial velocity in such a way the object remains in free fall until it lands back on the ground.

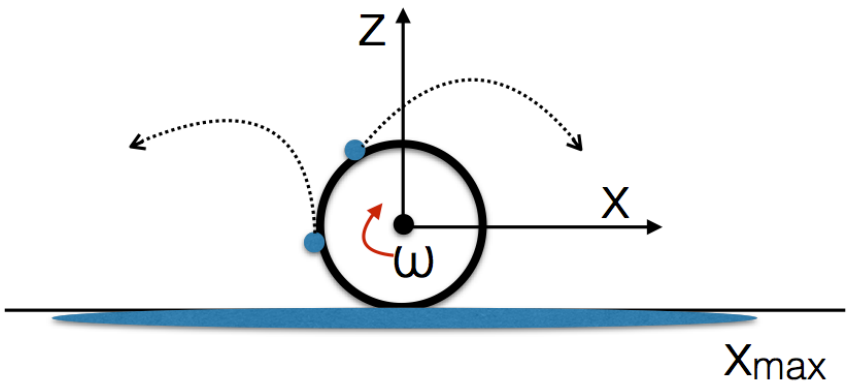
(a) *[10 pts.]* What must be the ratio of the initial velocity on the Earth to the initial velocity on the Moon such that the duration of the free fall is the same on Earth and on the Moon?

(b) *[10 pts.]* What must be the ratio of the initial velocity on the Earth to the initial velocity on the Moon such that the object culminates at the same height on Earth and on the Moon?

(V) *[10 pts.]* A bouncy ball is dropped vertically from a height of 3 m above ground. It bounces on the ground and goes up to 2 m above ground. Find the ratio between the velocity of the ball just before it hits the ground and just after it bounces.

(VI) *[10 pts.]* Randy Barnes holds holds the shot putter world record since 1990 with a throw of 23 m. Knowing that Randy's hand is 2.1 m above ground when he lets the shot go in a direction making a 45° angle with the horizontal, find the speed at which this champion can launch the 7.26 kg shot.

(VII) A wheel of radius R is uniformly rotating in a clockwise direction with angular velocity ω in the vertical $x-z$ plane as shown in the figure. The axis of the wheel about which it rotates is fixed at height R above the horizontal ground. The rim of the wheel is wet with water (which is constantly supplied to it) and small water drops are observed to fly off the points of the rim. The problem is most conveniently analyzed when the origin of the coordinate system is at the center of the wheel, right on the axis of rotation. A point M of the rim is specified by the angle θ counted clockwise between the x-axis and the direction from the center of the wheel to the considered point M. Gravitational acceleration has a magnitude g and is directed toward the negative values of z.



Taken from a problem composed by O.Starykh

(a) *[2 pts.]* Express the position vector $\vec{r}(\theta)$ of point M from the center of the wheel in the $\{\hat{x}, \hat{z}\}$ basis.

(b) *[2 pts.]* Express the velocity $\vec{v}(\theta)$ of the point M of the wheel.

(c) *[6 pts.]* Consider a droplet of water departing from the rim of the wheel at point M with an initial velocity $\vec{v}(\theta)$. As soon as the droplet is detached it uniformly accelerates downward (towards the negative z) because of gravitation. Find the coordinate x_I of the point where the droplet impacts the ground (in $z = 0$).

(VIII) *[10 pts.]* Consider the same system as in the previous problem except that the wheel is now in a horizontal plane at a distance h above ground and spinning around its vertical axis. Find the distance from the axis of the wheel where the droplets impact the ground.

(IX) *[10 pts.]* Consider an object in free fall with a uniform acceleration g. At a given time the object has a speed $v(t)$ and is at a height above ground $h(t)$. We define the quantity $Q(t) = \frac{v^2(t)}{2} + g \cdot h(t)$. Calculate the rate of change of $Q(t)$.

(X) *[10 pts.]* A roof makes a 30° angle with horizontal. A ball is released from rest on the roof. It rolls down over a distance of 5 m and then drops to the ground 7 m below. Find the horizontal distance between the point where the ball hit the ground and the house under the roof.