Homework 6

(I) Given $\vec{A} = 2\hat{x} - 3\hat{y} + 7\hat{z}$ and $\vec{B} = 5\hat{x} + \hat{y} + 2\hat{z}$ with $\hat{x}, \hat{y}, \hat{z}$ and orthonormal basis ($\hat{x} \cdot \hat{x} = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z} = 1$ while $\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = \hat{y} \cdot \hat{z} = 0$).

(a) *[1 pts.]* Find $\vec{A} + \vec{B}$.

(b) *[1 pts.]* Find $\vec{A} - \vec{B}$.

(c) *[2 pts.]* Find $|\vec{A}|$.

(d) *[2 pts.]* Find $\vec{B} \cdot \vec{B}$.

(e) *[2 pts.]* Find $\vec{A} \cdot \vec{B}$.

(f) *[2 pts.]* Find the cosine of the angle between $\vec{A}$ and $\vec{B}$.

(II) *[10 pts.]* Show that if $|\vec{A} - \vec{B}| = |\vec{A} + \vec{B}|$, then $\vec{A}$ and $\vec{B}$ are perpendicular.

(III) *[10 pts.]* $\hat{A}$ and $\hat{B}$ are unit vectors making angles θ_A and θ_B with the x-axis. By calculating $\hat{A} \cdot \hat{B}$ express $\cos(\theta_A - \theta_B)$ in terms of $\cos\theta_A$, $\cos\theta_B$, $\sin\theta_A$, and $\sin\theta_B$

(IV) *[10 pts.]* With $\{\hat{x}, \hat{y}, \hat{z}\}$ the basis of a cartesian coordinate system, $\vec{A} = 3\hat{x} + 4\hat{y} - 4\hat{z}$. Find the unit vectors in the (x, y) plane that are perpendicular to $\vec{A}$.

(V) *[10 pts.]* With $\{\hat{x}, \hat{y}, \hat{z}\}$ the basis of a cartesian coordinate system, $\vec{A} = \hat{x} + \hat{y} - \hat{z}$ and $\vec{B} = 2\hat{x} + \hat{y} - 3\hat{z}$. Find the two unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$.

(VI) A two dimensional cartesian coordinate system has a unit vector basis $\{\hat{x}, \hat{y}\}$. The unit vectors $\hat{x}'$ and $\hat{y}'$ are the images of $\hat{x}$ and $\hat{y}$ by a rotation of angle θ.

(a) *[3 pts.]* Express $\hat{x}'$ and $\hat{y}'$ in terms of $\hat{x}$, $\hat{y}$ and θ.

(b) *[3 pts.]* By manipulating your answer to the previous question, obtain the expressions of $\hat{x}$ and $\hat{y}$ in terms of $\hat{x}'$, $\hat{y}'$ and θ.

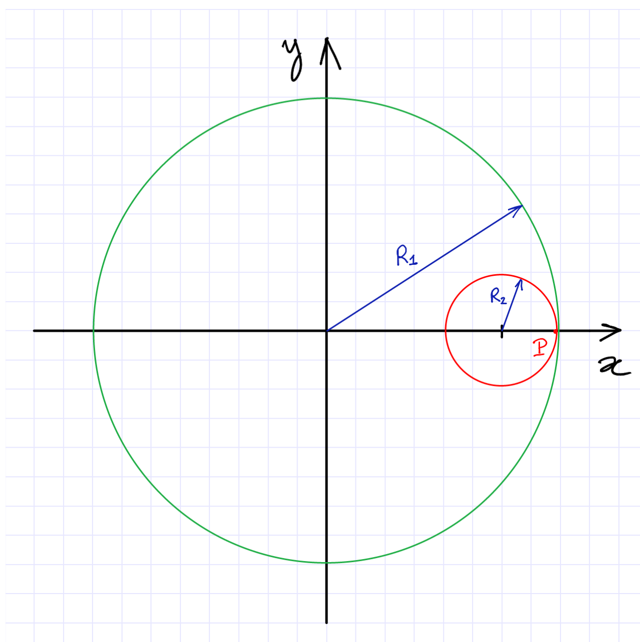
(c) *[4 pts.]* A vector $\vec{A} = A_x\hat{x} + A_y\hat{y}$ is given. Express the x' and y' components of $\vec{A}$.

(VII) *[10 pts.]* We have $\{\hat{x}, \hat{y}, \hat{z}\}$ the basis of a cartesian coordinate system and we are considering a vector $\vec{A}$ that makes an angle $\frac{\pi}{2} - \theta$ with $\hat{z}$ (so θ really is the angle $\vec{A}$ makes with the (x, y) plane) and whose perpendicular projection $\vec{A}_P$ on the (xy) plane makes an angle ϕ with $\hat{x}$. Express the components of $\vec{A}$ in the $\{\hat{x}, \hat{y}, \hat{z}\}$ basis in terms of the angles θ and ϕ, and A, the magnitude of $\vec{A}$.

(VIII) *[10 pts.]* Consider the Earth as a perfect sphere of radius R. A point on the surface of the Earth is specified by its longitude l_1 and latitude h_1. Another point is specified by l_2 and h_2. Find the expression of the distance between the two points along the geodesic connecting them. On a sphere, geodesics between two points are sections of great circles formed by the intersection between the plane containing the center and the two points and the considered sphere.

(IX) *[10 pts.]* A wheel of radius R rolls in a straight line along the x axis without slipping. Its center moves with a constant velocity V. The z axis measures vertical distance from the ground. A given point P of the wheel touches the road ($z = 0$) in $x = 0$ at time $t = 0$. Find the position $\vec{r}$, velocity $\vec{v}$ and acceleration $\vec{a}$ of point P as a function of time.

(X) A cylinder of radius R_2 rolls without slipping on the inside surface of a cylinder of radius R_1. The smaller cylinder moves in a counterclockwise direction and rolls around inside the other in a time period T. At time $t = 0$ the point P of the smaller cylinder is in contact with the larger cylinder on the x-axis as on the figure. (This might make you think of the spirograph of you childhood.)



A cylinder of radius R_2 rolling with out slipping on the inside of another of radius R_1 goes all the way around in a time T

(a) *[4 pts.]* Find the vector position $\vec{r}(t)$ of point P with respect to the point O at the center of the larger cylinder.

(b) *[3 pts.]* Find the velocity $\vec{v}(t)$ of point P.

(c) *[3 pts.]* Find the acceleration $\vec{a}(t)$ of point P.