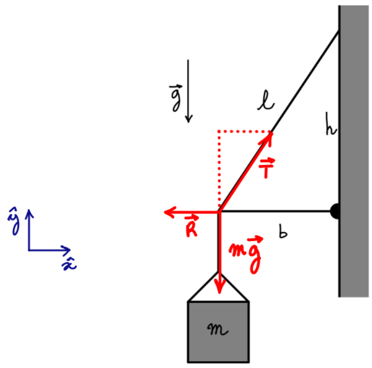
Midterm 2

(I) A mass m is hanging at the end of a horizontal beam of length b whose end is supported by a cable of length l. The horizontal beam is elastic with a spring constant k. Both the beam and the cable are secured to the same vertical wall, one vertically above the other at a distance h as indicated on the figure. The beam is attached to the wall via an articulate bracket. If it were not for the cable, the beam would be hanging down.



(a) *[25 pts.]* Find the tension of the cable.

Equilibrium condition $m\vec{g} + \vec{T} + \vec{R} = 0$

The y component of the equation gives $-mg + T_y = 0$

Using similar triangle $T_y = \frac{h}{\ell} T$ so $-mg + \frac{h}{\ell} T = 0$

so we find $\boxed{T = \frac{\ell m g}{h}}$

(b) *[25 pts.]* Find the change in length of the beam as a result of the stress due to the hanging mass. Clearly indicate if the length of the beam is increased or decreased.

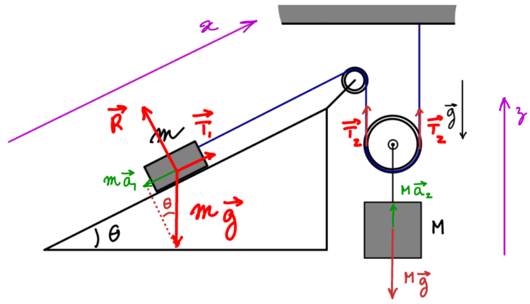
- The x component of the equilibrium equation gives $T_x + R_x = 0$

- Using similar triangles again $T_x = \frac{d\,T}{\ell} = \frac{d}{\cancel{\ell}}\frac{\cancel{\ell}\,m\,g}{h}$ so $R_x = -\frac{d\,m\,g}{h}$

- Following Newton's third law the beam is compressed by a force $\frac{d\,m\,g}{h}$

$\boxed{\text{so the beam is shorter by } \frac{d\,m\,g}{k\,h}}$

(II) *[35 pts.]* Consider the contraption represented by the figure. The kinetic friction coefficient between the ramp and the block is μ.



(a) *[40 pts.]* Find the acceleration of the block of mass m

- We may write Newton's second law for each of the blocks:

$$m\vec{a}_1 = \vec{R} + m\vec{g} + \vec{T}_1 \quad \& \quad M\vec{a}_2 = 2\vec{T}_2 + M\vec{g}$$

Additionally $|\vec{T}_1| = |\vec{T}_2| = T$.

- Chosing the x and y axis as indicated on the graph, we may project the equation for block m on the x axis & the equation for block M on the z axis, we obtain:

$$m\, a_{1x} = T - m g \sin(\theta) \quad \& \quad M a_{2z} = 2T - Mg$$

- The string is not stretchable so $a_{1x} = -2\, a_{2z}$ so the equations are

$$\begin{cases} m\, a_{1x} = T - m g \sin(\theta) \\ -\frac{1}{2} M a_{1x} = 2T - Mg \end{cases}$$

Two equations, two unknowns ⇒ We are good.

Multiplying the first equation by 2 & subtracting the second, we find:

$$\left(2m + \frac{M}{2}\right) a_{1x} = \left(M - 2m \sin(\theta)\right) g \quad \& \text{ finally } \quad \boxed{a_{1x} = \frac{2M - 4m \sin(\theta)}{M + 4m}\, g}$$

The expression is dimensionally correct.

If $M = 0$ $a_{1x} = -\sin\theta\, g$ as expected.

If $m = 0$ $a_{1x} = +2g$ also as expected.

(b) *[10 pts.]* Find the tension of the rope running through the pulleys.

We found $T = m\left(a_{1x} + g\sin(\theta)\right)$ & using our result for a_{1x}

$$T = m\left(\frac{2M - 4m\sin(\theta)}{M + 4m} + \sin(\theta)\right) g$$

& cleaning up by bringing the two terms over the same denominator we find

$$\boxed{T = \left(2 - \sin(\theta)\right)\frac{M m}{M + 4m}\, g}$$