Phenomenology of coupling between charge- and spin-current in systems with SOI

So far we have considered a few microscopic mechanisms of converting charge current into spin current. We have divided them into 

intrinsice ones: SOI comes from the band structure,

and

extrinsice ones: spin-orbit coupling comes from the impurity potential.

However, regardless of the microscopic origin, the possible effects of SOI can be considered at purely phenomenological level. This way we cannot calculate values of various quantities, like the transport coefficients, but we can fully specify what phenomena are possible in principle, and how their magnitudes are related to the values of microscopic quantities (which can serve to determine the latter).

The main ingredient of such phenomenological description of the effects related to SOI is the coupling between charge current and spin current, which we will consider below.
Phenomenology with inversion symmetry

We have seen many times that the way electrons subject to SO(3) behave depends crucially on the presence or absence of the inversion symmetry. Thus it makes sense to consider these cases separately.

Loosely speaking, we cannot have axial vectors proportional to polar vectors in the presence of inversion (e.g., a current (polar vector) cannot create spin polarization (axial vector)).

To facilitate further discussion, let us introduce currents of "dimensionless" charge and spin:

\[ j^q = e \tilde{q}, \rightarrow \tilde{q} \text{ - particle number current.} \]

\[ j^s = \frac{1}{2} \theta^{3} \tilde{q} \rightarrow q^s \text{ - spin polarization current.} \]

We would like to find the relations between charge and spin currents in the presence of SO(3).

To this end, imagine there is no SO(3), but there are currents of $s_z = 1$ and $s_z = -1$ particles driven by electric fields and density gradients in order:

\[ \phi^z \rightarrow \theta \rightarrow \text{means no SO(3)} \]

\[ \phi^z = -D \frac{\partial n \pm}{\partial x} + \mu \text{E} n \pm, \text{ where} \mu \text{ is the mobility of the particles, and} D \text{ is their diffusion coefficient (both assumed the same for both spins, valid for small polar)} \]
These currents correspond to the net number and polarization currents

\[ q_x^{(0)} = q_x^+ - q_x^- = -D \frac{\partial n_0}{\partial x} - \mu E n_0, \quad n_0 = n_+ - n_- \]

\[ q_x^{(0)} = q_x^+ + q_x^- = -D \frac{\partial n}{\partial x} - \mu E_s s^2, \quad s^2 = n_+ - n_- \]

from polarization, dimensionless.

We note that we can have a spin current even in the absence of spin, if we have a flow of spin-polarized carriers.

Now let us put to use what we learned before, while considering intrinsic/extrinsic mechanisms, to couple \( q^{(0)} \)'s, and then argue what we will have obtained the most general form of coupling:

\[ q_y = \Theta q_x \]

\[ q_y = -\Theta q_x \]

\[ (\text{sign of } \Theta \text{ is not known and it should be explained why it is the same appearing in both equations).} \]

Note there is no \( q_z \) generated by \( x \)-currents polarized in \( z \)-direction.

What kind of coupling between the currents does this qualitative picture imply?
we get
\[ q_y^+ = q_y^- = -\Theta (q_x^+ + q_x^-) = -\Theta q_x^0, \]
\[ q_y = q_y^+ + q_y^- = -\Theta (q_x^+ - q_x^-) = -\Theta q_x^0. \]

or for the total quantities:
\[ q_x^0 = q_x^0, \quad q_y^0 = q_y^0 - \Theta q_x^0 z, \quad q_z = q_z^0, \]
\[ q_x^0 = q_x^0, \quad q_y^0 = q_y^0 - \Theta q_x^0 z, \quad q_z = q_z^0. \]

We can see that these relations are a particular
rotationally invariant definition
\[ q_i^0 = q_i^0 - \Theta \varepsilon_{i\delta k} q_k^0, \]
remove these
\[ q_{i\delta} = q_{i\delta}^0 + \Theta \varepsilon_{i\delta k} q_k^0 \]

It is clear that this is the most general way
to couple a vector \( q_i \) and a tensor \( q_{i\delta} \).
\( \varepsilon_{ij} \) is the only 2nd rank tensor available.
These relations are valid to 1st order in \( \Theta \).

Having coupled the charge and spin currents,
we can discuss the physical consequences
of such coupling.
Now we can write down the current densities:

\[ j^{(1)} = e \dot{\vec{r}} \quad \text{and} \quad j^{(2)} = -eD \frac{\partial n}{\partial t} - e\mu \vec{E} \nabla n + \theta D \cdot \vec{E} \cdot \vec{j} \cdot \nabla \frac{\partial \rho}{\partial t} + \theta \mu \vec{E} \cdot \vec{E} \times \vec{E} \cdot \vec{j} \nabla \frac{\partial \rho}{\partial t} \]

\[ j'_{\phi} = -eD \frac{\partial n}{\partial t} - e\mu \vec{E} \cdot \nabla n + \theta D \cdot \vec{E} \cdot \vec{j} \cdot \nabla \frac{\partial \rho}{\partial t} + \theta \mu \vec{E} \cdot \vec{E} \times \vec{j} \nabla \frac{\partial \rho}{\partial t} \]

*Physical effects contained in these equations.*

(2) $\theta \mu \vec{E} \times \vec{j} \nabla \frac{\partial \rho}{\partial t}$ — **Anomalous Hall effect**.

This needs finite spin polarization, which can exist either due to magnetization of a ferromagnet, or can be created optically.

$\theta \mu \vec{E} \times \vec{j} \nabla \frac{\partial \rho}{\partial t}$ — current created by curl of the magnetization. This is **Inverse Spin Hall effect** (see below).
\( \Theta \delta \delta (D \frac{\partial n}{\partial t} + \mu E_n n) \) - Spin Hall effect

In words, this shows that flux of electrons on \( x \)-direction created a flux of \( n \)-spins in \( y \)-direction. All three are mutually orthogonal. In x cylinder this happens.

The appearance of this current leads to spin accumulation at sample boundaries.

To see what is going on, consider the continuity equation for spin polarization:

\[ \frac{\partial S^i}{\partial t} + \frac{\partial}{\partial x^j} q^i_{\delta j} = - \frac{\partial c^i}{\partial x^i} \]

The geometry is shown here.

Then \( \frac{\partial}{\partial y^j} q^i_{\delta j} = - D \cdot \frac{\partial^2 S^i}{\partial x^j \partial x^j} \). (other terms do not have divergence)

In our geometry we obtain

\[ D \frac{\partial^2 S^i}{\partial y^2} = \frac{g^i_{\delta j}}{2s} \Rightarrow \frac{\partial^2 S^i}{\partial y^2} = \frac{1}{2s} g^i_{\delta j} \]

Boundary condition: is spin current through the boundary:

\[ \frac{\partial S^x}{\partial y} = 0, \quad \frac{\partial S^y}{\partial y} = 0, \quad - \frac{\partial S^z}{\partial y} = \Theta \mu E_x \Rightarrow \frac{\partial S^2}{\partial y} = - \frac{\Theta \mu E_x}{D} \]
Thus we see that only \( S^z(y) \) exists (\( S^y(z) \) may exist, too, by we assume to be far from the xy-boundaries).

Thus we get \( S^z(y) = S^z(0) e^{-\frac{y}{l_s}} \).

\[
-\frac{1}{l_s} S^z(0) = -\frac{\alpha e\hbar}{\beta} E_x \Rightarrow \frac{\alpha e\hbar}{\beta} E_x \exp\left( \frac{y}{l_s} \right)
\]

\[
l_s \approx \sqrt{\frac{2 \hbar\gamma}{e^2 F_{or}}} \approx \sqrt{\frac{2 \times 10^{-32}\text{J} \cdot \text{s}}{0.65 \times 12 \times 10^{-12} \text{V} \cdot \text{m}}} \approx 1 \mu\text{m}.
\]

(See a review at Kato's et al. experiment.)

The Inverse Spin Hall effect

The (direct) spin Hall effect leads to generation of a pure spin current in the presence of a transport current in the perpendicular direction.

The inverse SHE, well, the inverse of that: passage of a spin current can create a net charge current in a 1 direction.

**Direct SHE:**

\[
\begin{array}{c}
\circlearrowleft \\
\circlearrowright
\end{array}
\]

**Inverse SHE:**

\[
\begin{array}{c}
\circlearrowleft \\
\circlearrowright
\end{array}
\]

\[
\begin{array}{c}
\circlearrowright \\
\circlearrowleft
\end{array}
\]

spin current
charge current
How does one produce a pure spin current?

The basic idea can be illustrated:

\[ \psi_0 \xrightarrow{\mathbf{H}(x)} \psi(x) \]

Create non-equilibrium spin polarization, and let it diffuse. The spin polarization has to be \( \perp \) to the diffusion direction.

\[ m + n = \text{const} \rightarrow \text{no net charge current}. \]

\[ -D \frac{\partial m}{\partial x} - (-D \frac{\partial n}{\partial x}) = -D \frac{\partial S^z}{\partial x} \neq 0 \Rightarrow \text{curl } \mathbf{S} \neq 0 \Rightarrow \text{net charge current in } y \text{ direction}. \]

This can be achieved in a variety of ways:

1) Optical orientation (first experiment)

2) Electrical injection (Valenzuela, Michljevic 2006)

3) Spin pumping (Saitoh 06)
Phenomenology without inversion symmetry.

The main additional ingredient to the case with broken inversion symmetry is the possibility to generate spin polarization from a current (and vice versa). Indeed, the former is a pseudo-vector, while the latter is a polar (regular) vector, thus they have opposite properties wrt inversion. Therefore, a relation like

$$ S = M \hat{\mathbf{j}} $$

is possible only when inversion is broken.

Let us apply this logic to a 2DEG with Rashba effect:

$$ H = \frac{\mathbf{p}^2}{2m} + \lambda (\hat{\mathbf{z}} \times \hat{\mathbf{p}}) + U(\mathbf{r}) - e\mathbf{F} \cdot \mathbf{r} $$

where $U(r)$ is the disorder potential, and $\mathbf{E}$ is the external electric field.

We consider the case when $\lambda$ is small such that $d\mathbf{F} \ll \frac{1}{t_0}$, spin splitting of the band structure is much smaller than disorder broadening of Bloch states.

In this case we can solve the problem without $\lambda$, and consider it as a perturbation.
A detailed solution of the kinetic equation for a 2DEG will be given in Lecture 23. Here we use qualitative arguments.

In an electric field, electrons gain a drift velocity, determined by their instability:

\[ \vec{v}_d = \mu \vec{E}, \quad \mu = \frac{e}{m} \rightarrow \text{transport mean free time}. \]

Thus they have a drift momentum \( \vec{p}_d = e \vec{v}_d \vec{E} \).

In a state with a net momentum, the Rashba effect looks like an effective magnetic field

\[ \vec{B}_R = \frac{\hbar}{e} \left( \frac{\vec{n} \times \vec{E}}{2} \right) = \frac{e}{\hbar} \vec{e}_y \vec{E} \times \vec{e}_y \]

which polarizes electrons:

\[ \Sigma_j = 2 \langle \vec{B}_R^2 \rangle = 2 \left( \frac{m \nu}{2 \pi \hbar^2} \right) B_R \]

\[ \nu = \frac{\langle \vec{E}_F \rangle}{\hbar} = \frac{e \vec{E}_F}{2m} \]

Relative polarization:

\[ \frac{\Sigma_j}{\hbar} = \frac{m \nu^2}{2 \pi \hbar^2} e \vec{E}_F \times \vec{E} \times \frac{e \vec{E}_F}{2m} \]

\[ \frac{\Sigma_j}{\hbar} = \frac{m \nu^2}{2 \pi \hbar^2} \frac{e \vec{E}_F \times \vec{E} \times e \vec{E}_F}{2m} \]

Each of these factors is small, so the net polarization is rather small, too.