Triangular lattice antiferromagnets -- open questions

Oleg Starykh, University of Utah

Leon Balents, KITP
Hosho Katsura, U Tokyo
Jason Alicea, UC Irvine
Andrey Chubukov, U Wisconsin
Christian Griset, UCSB
Shane Head, U Utah

Disentangling quantum many-body systems, KITP, November 11, 2010

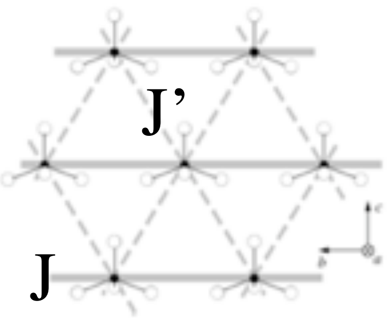
Outline

- Motivation (Cs_2CuBr_4) and theoretical progress
  - crucial role of spatial anisotropy
    - Cs_2CuCl_4 and Cs_2CuBr_4
    - organic materials (t, t', U...)
- Phase diagram of spatially anisotropic Heisenberg model
  - Large-S analysis of interacting spin waves near J'=J
  - Approach from one dimension, J' << J
- Open questions

Experiment: M=1/3 magnetization plateau in Cs_2CuBr_4

★ Observed in Cs_2CuBr_4 (Ono 2004, Tsuji 2007) **J'/J = 0.5-0.75**
but not Cs_2CuCl_4 [**J'/J = 0.34**]

S=1/2



140 J. Phys. Soc. Jpn. Vol. 74 (2005) Supplement

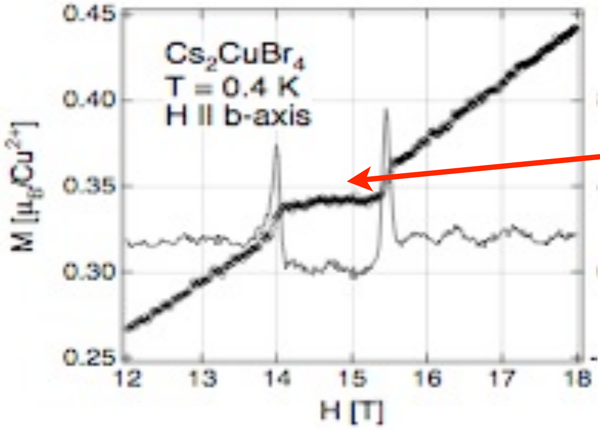
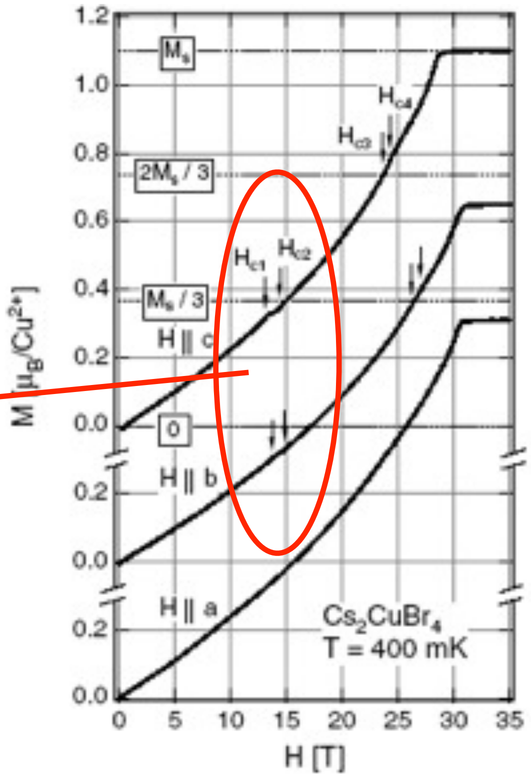
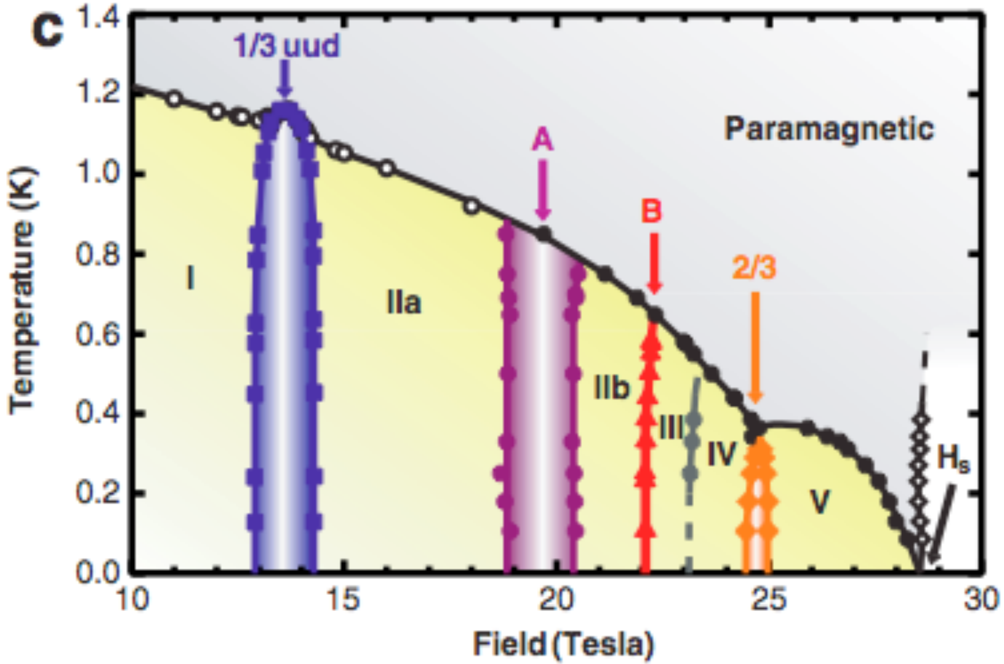


Fig. 8. The magnetization curve and dM/dH versus I
measured at $T = 0.4$ K in magnetic fields up to 20 T

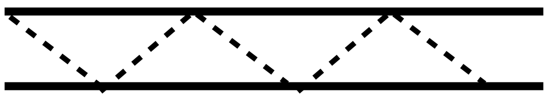




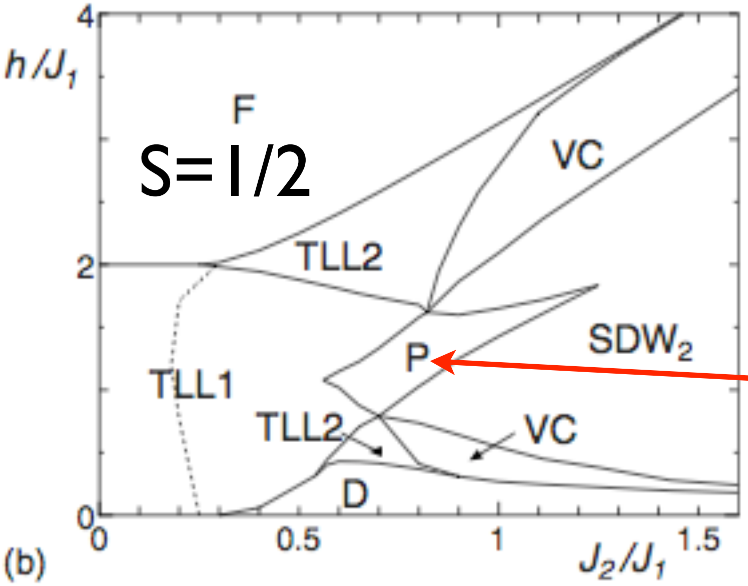
9 experimental phases
vs
3 theoretical

Fortune et al, Phys. Rev. Lett. 102, 257201 (2009)

Progress in one dimensional J_1-J_2 chain (zig-zag ladder)



Okunishi, Tonegawa JPSJ (2003)
Hikihara et al PRB (2010)



S=1/2

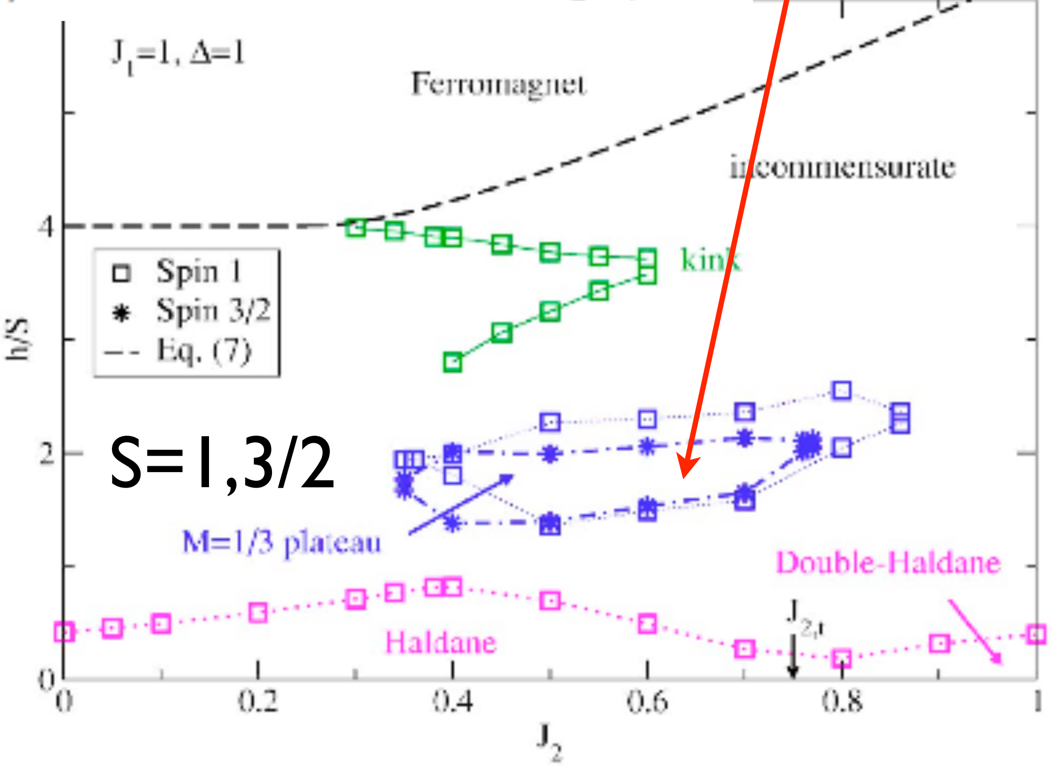
Common feature: robust ***plateau***;
expect more of it in D=2

M=1/3 plateau

agrees with Oshikawa, Yamanaka, Affleck argument (PRL 2007):

$$p\,S\,(1 - M) = \text{integer}$$

p = period, S = spin,
M = magnetization:
M=1/3, p=3
possible for all S



S=1,3/2

Heirich-Meisner et al PRB (2007)

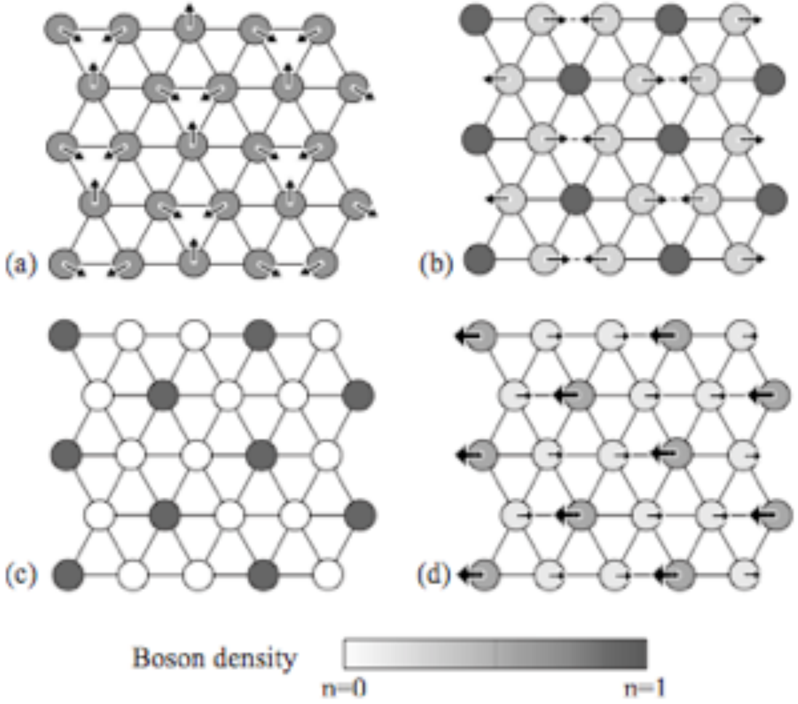
……

plateau is centered around $J_2 = J_1/2$ point for S > 1/2; semi-classical spin wave expansion is possible there (OS 2009)

D = 2

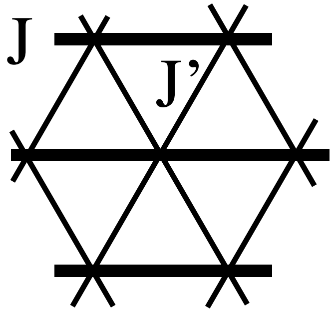
- surprisingly complex phase diagram of spatially anisotropic triangular lattice antiferromagnet
  - no definite conclusions from numerical studies yet...
- connections with interacting boson system
  - Superfluids
  - Mott insulators
  - Supersolids

Nikuni, Shiba 1995
Heidarian, Damle 2005
Wang et al 2009
Jiang et al 2009
Heidarian, Sorella, Becca 2009
Tay, Motrunich 2010

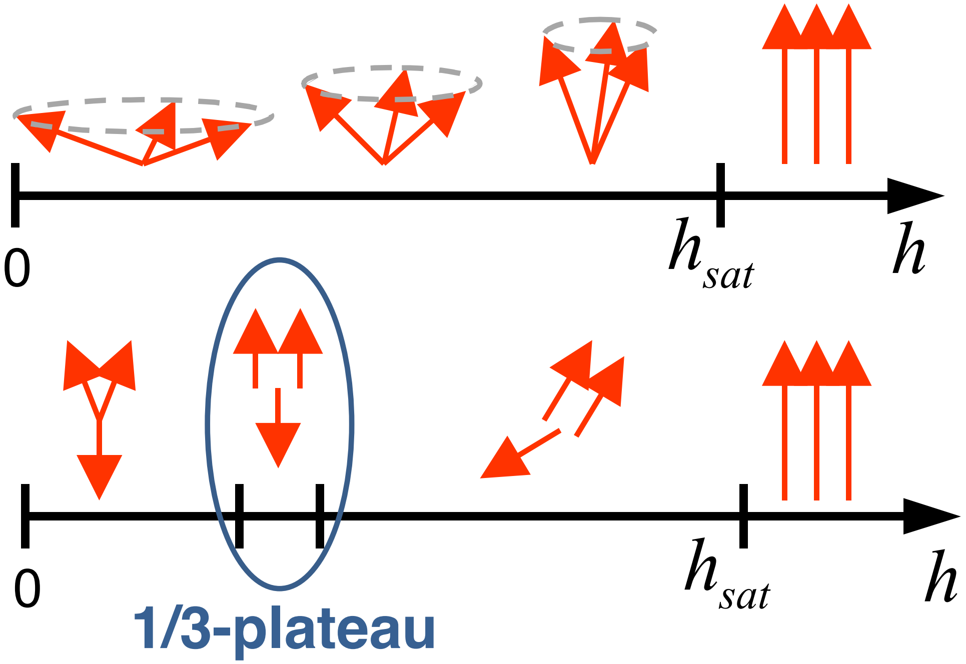


Spatially anisotropic model near J’ = J

$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



$$J' \neq J$$



Umbrella state: favored classically; energy gain (J-J’)^2/J

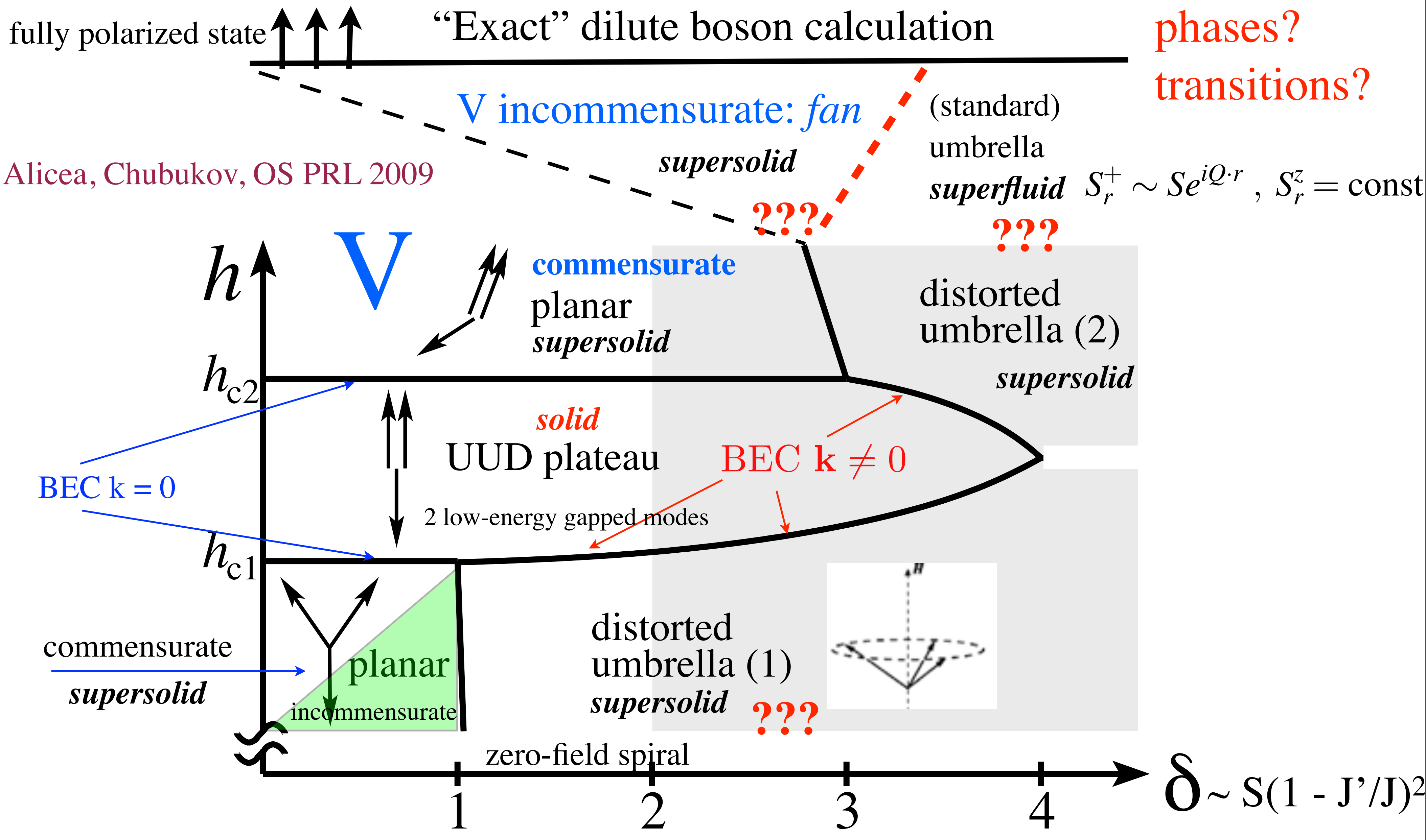
Planar states: favored by quantum fluctuations; energy gain J/S

The competition is controlled by dimensionless parameter $\delta = S(J - J')^2/J^2$

- Technical formulation: spatial anisotropy J-J’ causes softening of interacting (including 1/S correction) spin waves

Sketch of phase diagram (J-J' << J)

Questions:
phases?
transitions?



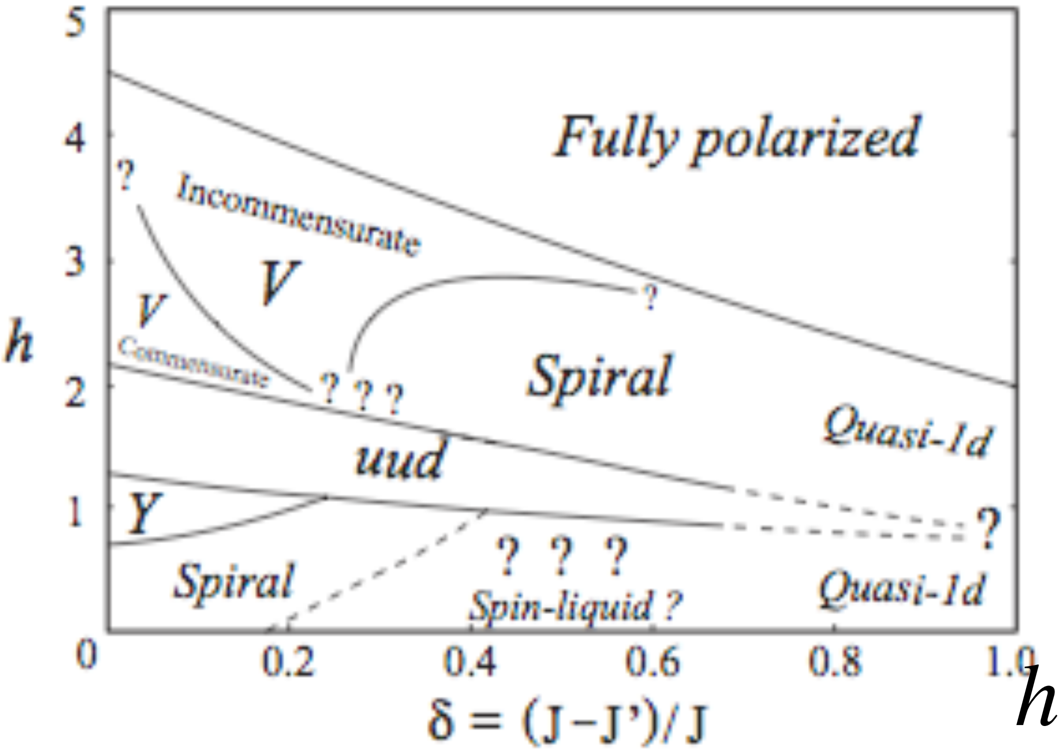
Alicea, Chubukov, OS PRL 2009

supersolid: $S_r^+ \sim S\cos[Q\cdot r]$, $S_r^z \sim S - S\cos^2[Q\cdot r]$

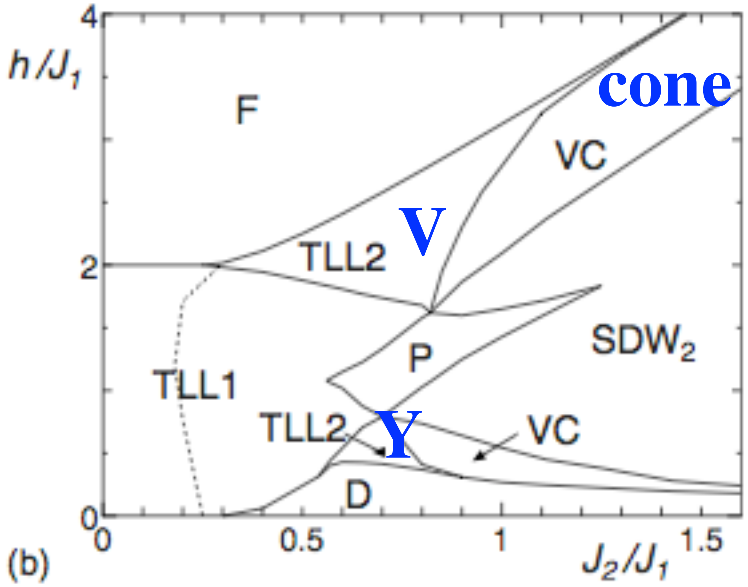
Comparison with numerics

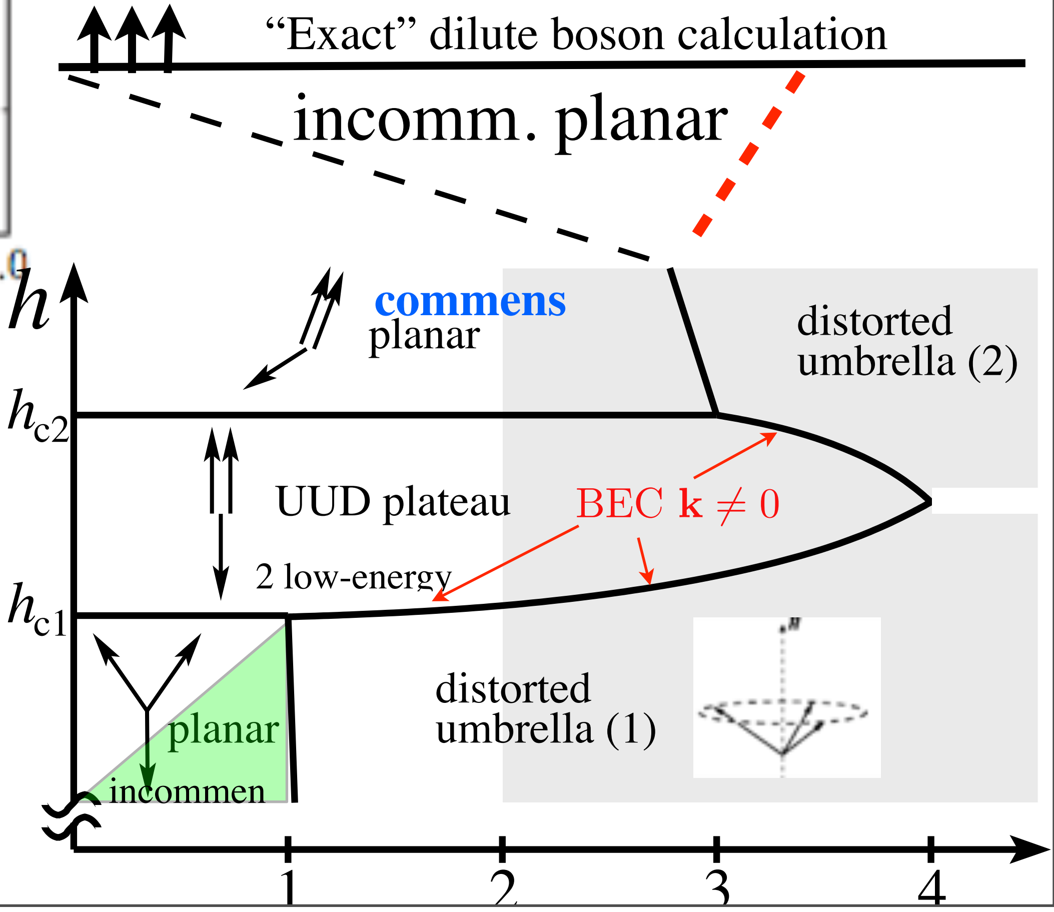
Variational wave function calculation

Tay, Motrunich PRB 2010



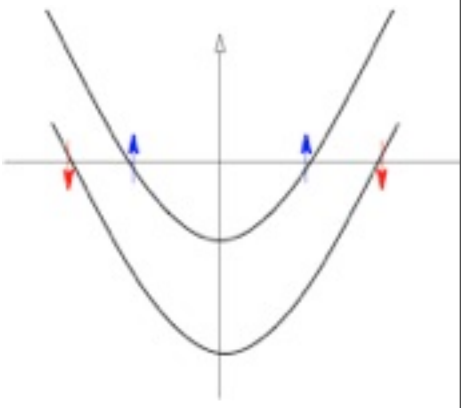
J_1-J_2 chain DMRG





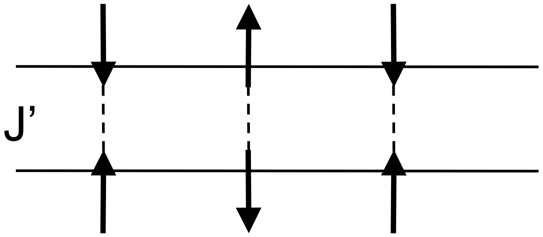
Phase diagram for smaller J'/J?

J’ << J: weakly coupled Heisenberg chains in magnetic field



- non - frustrated inter-chain coupling $\vec{S}_r \cdot \vec{S}_{r'} \rightarrow N_r^x N_{r'}^x + N_r^y N_{r'}^y + N_r^z N_{r'}^z$

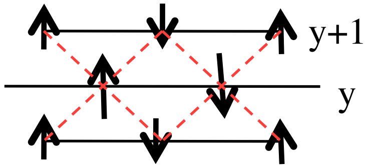
most relevant   less relevant



$2\pi R^2 < 1/(2\pi R^2)$

spins order in the plane perpendicular to the direction of magnetic field (z): **umbrella / cone / spin-flop states**

- frustrated inter-chain coupling



$$\vec{S}_{x,y}\cdot(\vec{S}_{x,y+1}+\vec{S}_{x+1,y+1}) \rightarrow \underbrace{N_y^x \partial_x N_{y+1}^x + N_y^y \partial_x N_{y+1}^y}_{\text{less relevant}} + \sin(\delta) S^z_{\pi-2\delta}(y) S^z_{\pi+2\delta}(y+1)$$

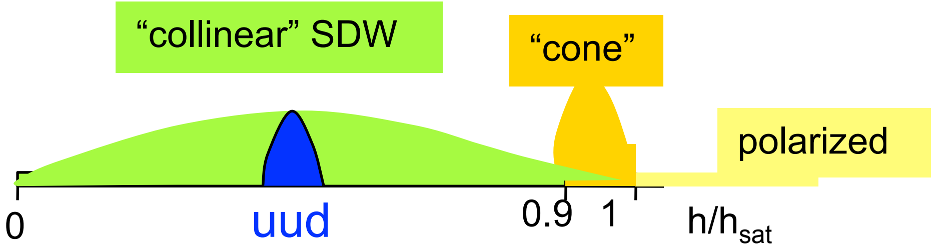
most relevant (small to intermediate fields)

$1+2\pi R^2 > 1/(2\pi R^2)$

★ frustration promotes collinear **SDW** order

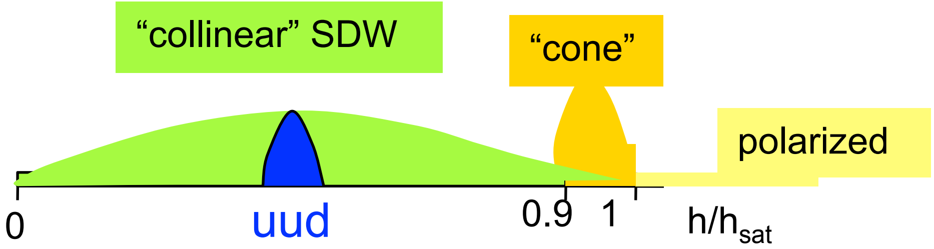
J’ << J limit

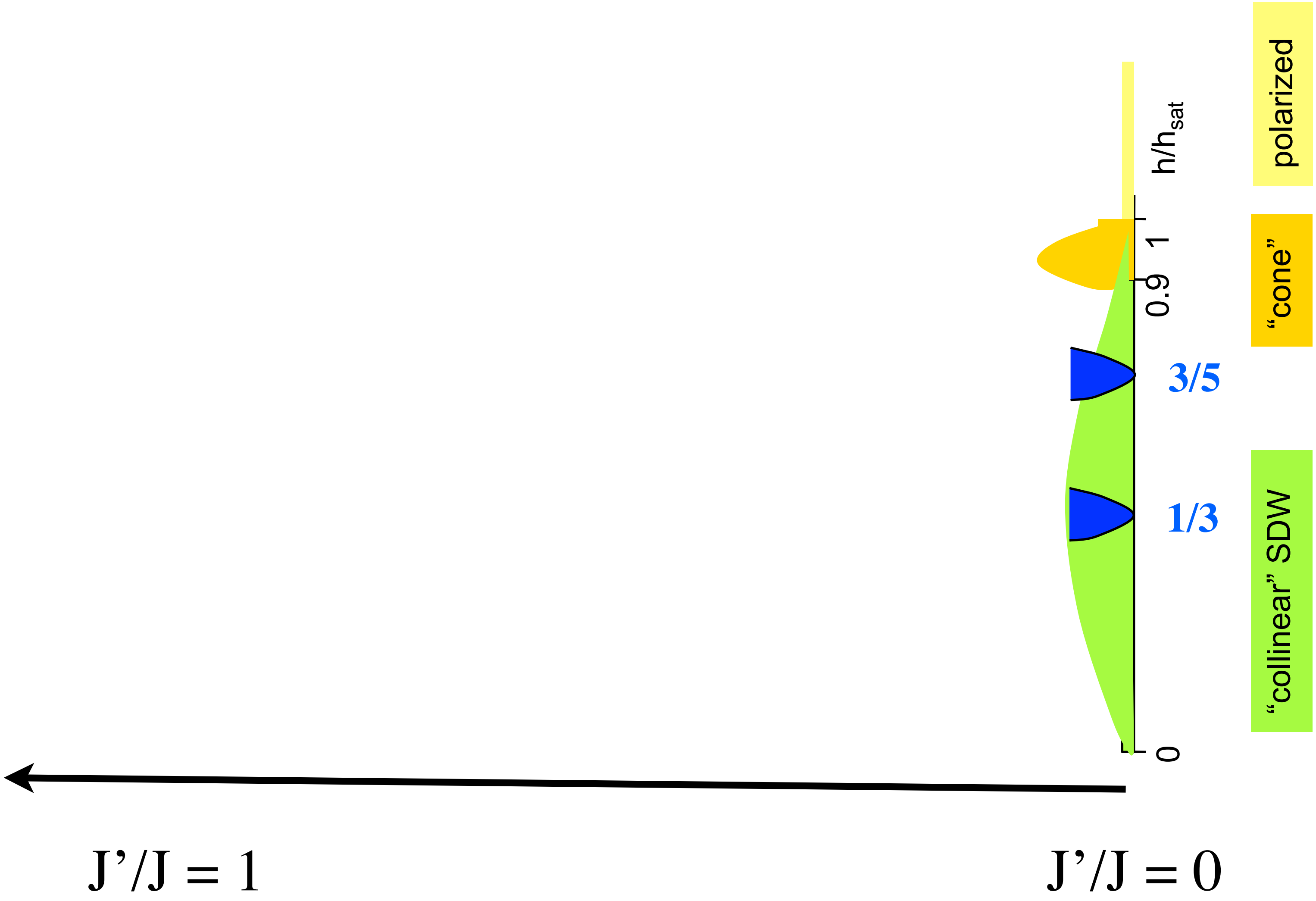
Katsura, OS, Balents PRB 2010



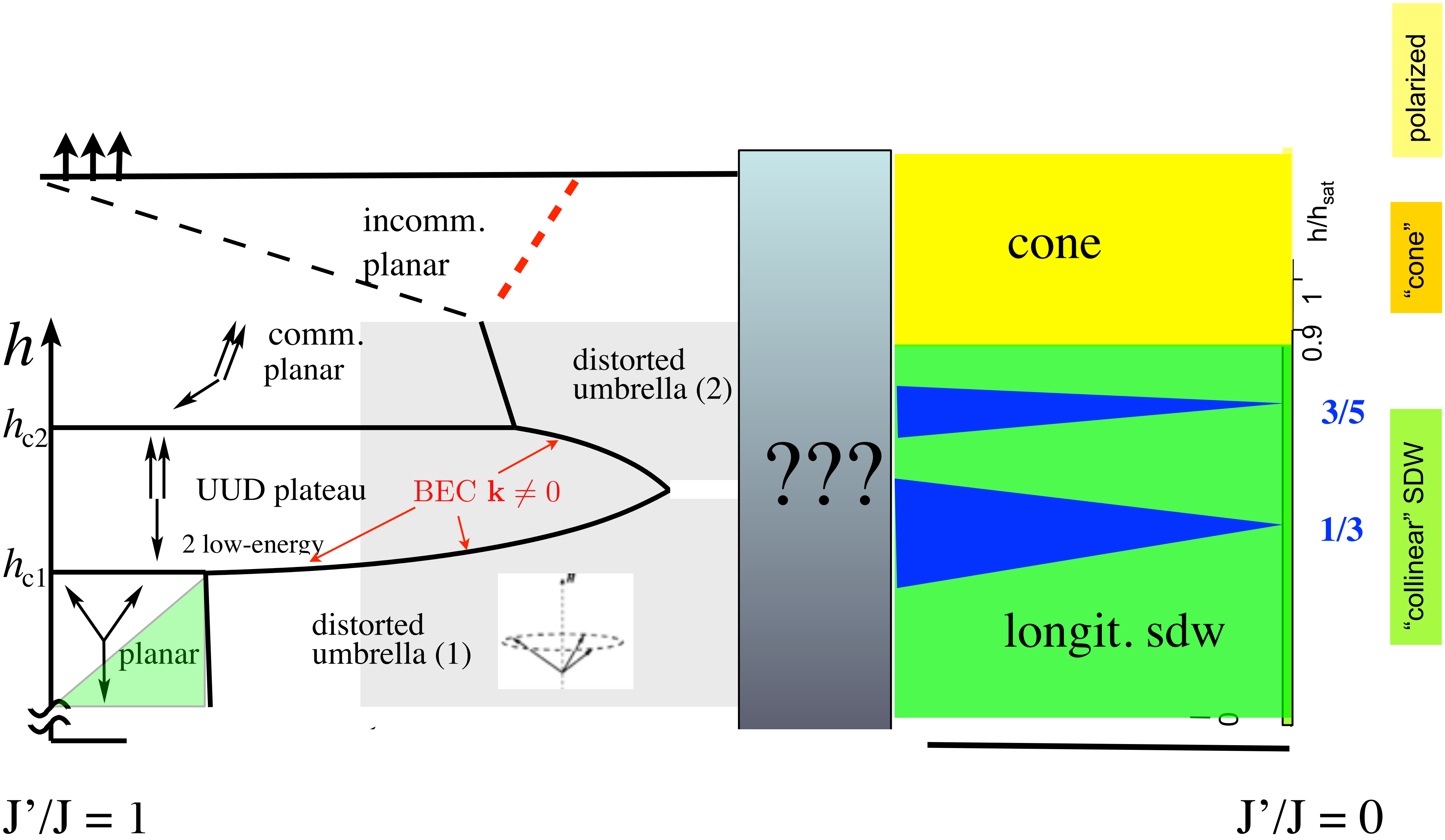
J’ << J limit

Katsura, OS, Balents PRB 2010



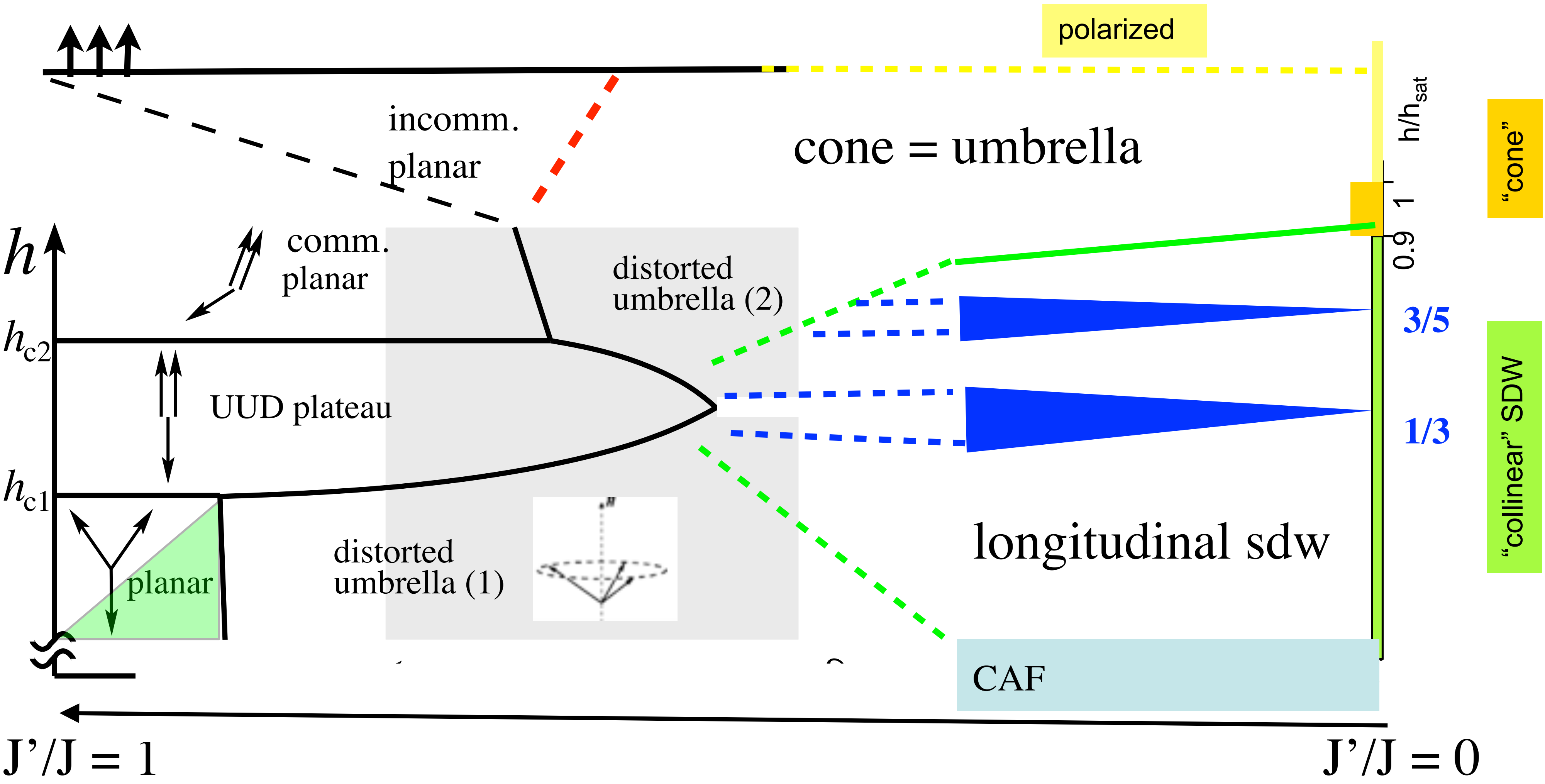


J’ << J limit to J’ = J point...



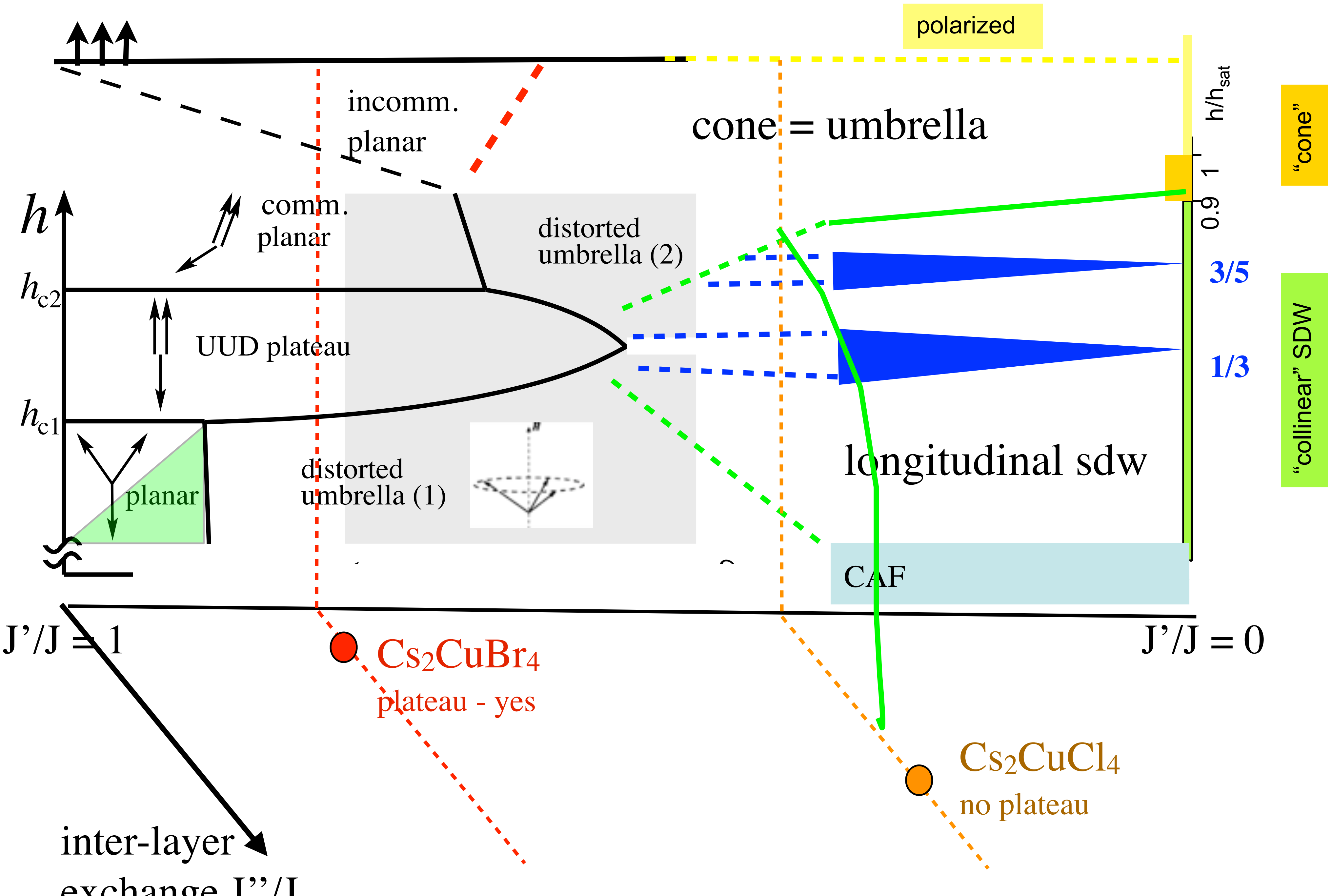
Global phase diagram

Hypothesis: 1/3 plateau extends for all 0 < J'/J < 1;
other magnetization plateaux terminate above some critical J'/J ratio.



Question: how many phases are there?
magnetization plateaux?

Experimental relevance



polarized

incomm.
planar

cone = umbrella

h/h_{sat}

1

0.9

“cone”

h

comm.
planar

distorted
umbrella (2)

h_{c2}

3/5

UUD plateau

1/3

h_{c1}

“collinear” SDW

longitudinal sdw

planar

distorted
umbrella (1)

CAF

J’/J = 1

Cs_2CuBr_4

plateau - yes

J’/J = 0

Cs_2CuCl_4

no plateau

inter-layer
exchange J’’/J

large J’’/J favors classical cone order !

Conclusions

- ★ Magnetization plateau persists for all J’/J (**?**)
  - – semiclassical interacting spin waves near J - J’ << J
  - – 1d scaling + symmetry arguments near J’ << J
- ★ Many interesting *magnetically ordered* phases
  - \- global phase diagram of triangular antiferromagnet ?
  - – Longitudinal SDW (**?**)
    - S=1/2 vs S=1 (?)
  - – plateau for ferromagnetic J_1**?** [$LiCuVO_4$]
- ★ Many open experimentally relevant questions, excellent problem for numerical studies