Physics 2220 – Spring 2018	**EXAM 1**	Dr. Eugene Mishchenko
Name: Solution	Unid:	**1**
Discussion TA:	Discussion Section:	

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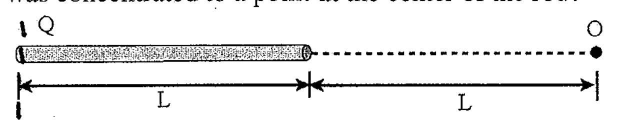
Consider a thin rod of length L uniformly charged along its length with the net charge Q.

(a) What is the magnitude of the electric field produced by the rod at a point O located at distance L from its end, as shown in the picture?

(b) What is the magnitude of the electric force on an electron situated at point O if L = 25cm and Q = 10 μC?

(c) How would your answer to (a) change if all of the charge Q was concentrated to a point at the center of the rod?



~~x=0~~ $x=-2L$ ~~x=2L~~ $x=0$

$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$, all one dimension

$E = \int \frac{k\,dq}{r^2}\hat{x}$, $dq = \lambda dx = \frac{Q}{L}dx \Rightarrow E = \frac{kQ}{L}\int_{-2L}^{-L}\frac{dx}{x^2}$

$E = \frac{kQ}{L}\left[-\frac{1}{x}\right]_{-2L}^{-L}$ so $E = \frac{-kQ}{L}\left(\frac{1}{-L} - \frac{1}{-2L}\right) = \boxed{\frac{kQ}{2L^2}}$

if $L = 0.25\,m$ and $Q = 10^{-5}\,C$, $\boxed{E = 720\,\frac{kN}{C}}$ and $\boxed{F = 1.2\times10^{-13}N}$

Point charge electric field $E = \frac{kq}{r^2}$, $r^2 = (L+\frac{L}{2})^2 = \frac{9}{4}L^2$

so $\boxed{E = \frac{4kQ}{9L^2}}$ is slightly less than the rod.

Rubric

(a) 6 pts

(b) 3 pts

(c) 6 pts

Solutions

Physics 2220 – Spring 2018 | **EXAM 1** | Dr. Eugene Mishchenko

Name:	Unid:	2
Discussion TA:	Discussion Section:	

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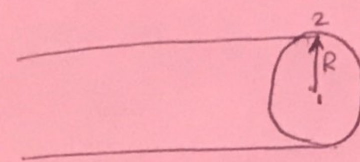
An infinitely long cylinder of radius R is uniformly charged with the volume charge density $\rho < 0$.

(a) What is the difference of the electric potential produced by the cylinder between a point on its axis and a point on its surface?

(b) Which point has a higher potential?



(a) potential difference between 2 points :-

$$\Delta V = -\int_{R_1}^{R_2} \vec{E} \cdot ds = -\int_{0}^{R} \vec{E} \cdot dr.$$

$$\vec{E} = \frac{Q}{\varepsilon_0 \cdot A} \text{ (from Gauss's law).}$$

$$Q = \int_{R_1}^{R_2} dq = \int_{R_1}^{R_2} \rho \, dV = \int_{0}^{R} \rho \cdot (2\pi r h) \, dr = \rho \pi R^2 h.$$

$$\therefore \vec{E} = \frac{Q}{\varepsilon_0 A} = \frac{\rho \pi \cancel{R}^{\cancel{2}} \cancel{h}}{\varepsilon_0 \cdot 2\cancel{\pi} \cancel{R} \cancel{h}} = \frac{\rho R}{2\varepsilon_0}.$$

$$\therefore \Delta V = -\int_{0}^{R} \frac{\rho R}{2\varepsilon_0} \, dr. \boxed{= -\frac{\rho R^2}{4\varepsilon_0}.}$$

(b) Since $\Delta V \propto R^2$, the larger the distance the higher the potential. Hence The point on the surface has higher potential

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Physics 2220 – Spring 2018 | EXAM 1 | Dr. Eugene Mishchenko

3

Name:	Unid:
Discussion TA:	Discussion Section:

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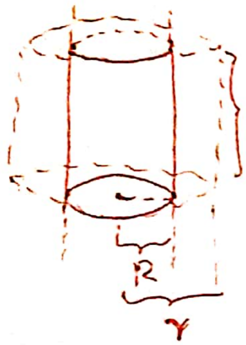
Solution

An infinitely long cylinder of radius R is charged with the axially symmetric volume charge density $\rho(r)$ that depends on the distance r to the axis.

(a) What is the electric field E(r) at an arbitrary distance r from the axis?

(b) If the charge density were given by the expression $\rho(r) = A \sin(\pi r/R)$, for r < R, and $\rho(r) = 0$ for r > R, what would be the value of the electric field at the surface of the cylinder r = R?

a)



$dq = \rho(r)\,dV$

Using a Cylindrical Gaussian Surface (arbitary distance r)

$dV = 2\pi r L\,dr$

Using Gauss law,

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \cdot 2\pi r L = \frac{\int_0^r dq}{\varepsilon_0} = \frac{\int_0^r 2\pi r \rho(r) L\,dr}{\varepsilon_0}$$

$$\boxed{E = \frac{\int_0^r \rho(r)\, r\,dr}{r\,\varepsilon_0}} \quad — ①$$

(b) for $r < R \Rightarrow \rho(r) = A\sin(\pi r/R)$

Plug this into equation ① and integrate from $r=0$ to $r=R$

$$E(R) = \frac{\int_0^R A\sin(\pi r/R)\, r\,dr}{R\,\varepsilon_0}$$

using integration by parts,

$$E(R) = \frac{A}{R\varepsilon_0}\int_0^R r\left(\frac{-R}{\pi}\right)\frac{d\cos\frac{\pi r}{R}}{dr}\,dr = \frac{A}{R\varepsilon_0}\left[\frac{-R}{\pi}\, r\cos\left(\frac{\pi r}{R}\right)\Big|_0^R + \frac{R}{\pi}\int_0^R \cos\left(\frac{\pi r}{R}\right) dr\right]$$

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$$\therefore E_{(R)} = \frac{A}{R\varepsilon_0}\left[-\frac{R}{\pi}\Big(R\underbrace{\cos\pi}_{-1} - 0\Big) + \left(\frac{R}{\pi}\right)^2\underbrace{\big(\sin\pi - \sin 0\big)}_{0}\right]$$

$$\therefore \boxed{E_{(R)} = \frac{AR}{\pi\varepsilon_0}}$$

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